

Heralded generation of the Optical Schrödinger Cat State

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Using a simple optical setup I show how optical entanglement between a macroscopic and a microscopic state - the so-called Schrödinger Cat state or micro-macro state - can be generated. The entangled state is heralded and is thus produced a priori in contrast to previous proposals. A simple variant of the scheme allows for the preparation of the entangled state at distant sites connected with a lossy link. Furthermore, I show that the state can be used to map a microscopic qubit onto a macroscopic one thereby linking the qubit processor with a qumode processor.

Quantum superpositions are at the heart of quantum mechanics. Simple examples are two-dimensional superpositions of microscopic systems such as two-level atoms, polarization of a single photons and the spin of an electron. Being at a microscopic level, these superpositions are readily accepted but if they are brought to the macroscopic level they become counterintuitive and hardly imaginable. This is commonly illustrated by the famous gedanken experiment of Schrödinger in 1935 where he considers the superposition of a cat in two distinct states; dead and alive [1]. In this experiment the cat is entangled with a microscopic degree of freedom, namely the discrete energy levels of an atom. Therefore, the proposal does not only demonstrate the superposition principle on a macroscopic scale but also the peculiar feature of nonlocality [2].

In recent years there has been a strong focus on bringing quantum mechanics into a macroscopic realm through careful state engineering and suppression of environmental noisy modes [3]. Macroscopic superpositions of atomic clouds [4], superconducting circuits [5, 6], ions [7] and microwaves [8] have been prepared, and there are proposals on how to push this into a regime of massive systems [9] and even living organism [10].

In the pure optical regime there has also been a number of successful attempts to generate a macroscopic quantum states. One example is the generation of coherent state superpositions by means of photon subtraction of a squeezed vacuum state [11–14]. Although being useful for quantum information processing [15], strictly speaking, these states are not cat states in the spirit of Schrödinger as the macroscopic states (here coherent states) are not entangled with a microscopic degree of freedom. Another realization of a macroscopic state is the so-called micro-macro state in which the polarization degree of freedom of a single photon is entangled with distinct states containing a large number of photons [16]. These states have been produced in a non-heralded fashion [17, 18] and their characterization has been discussed in several papers [18–21].

In this paper I suggest a number of different ways to generate a heralded optical cat state using standard quantum optical tools. As opposed to the previous pro-

posals and experiments on micro-macro entanglement, in the present scheme the entanglement is produced between a microscopic particle photon number degree of freedom (zero or one photon) and a macroscopic wave degree of freedom also known as a qumode [22]. As a result of the heralded generation, the state can be fully characterized with homodyne tomography. I also discuss how this state can be used to map a microscopic qubit onto a macroscopic qumode by means of teleportation.

In the experiment of De Martini et al [18], the polarization degree of freedom of a single photon was entangled with the polarization degree of freedom of a macroscopic state containing a large number of photons exceeding 10^4 . This was enabled by unitary amplification (using a phase-insensitive, polarization nondegenerated two-mode squeezer) on one half of a polarization entangled photon pair produced by parametric downconversion. The experiment was carried out in the coincidence basis, and thus the resulting micro-macro entangled state was generated a posteriori: Although the amplification process is deterministic, the generation of polarization entangled photons was non-heralded. Herald generation of polarization entangled photons have recently been realized [23, 24] in complicated setups and its extension to produce a heralded micro-macro state renders the setup even more challenging.

Instead of amplifying polarization entangled photons with a two-mode squeezer, I suggest to amplify a path-entangled single photon with a single-mode squeezer. The setup is shown in Fig. 1a. A single photon is prepared (e.g. by heralded parametric down conversion) and subsequently split on a balanced beam splitter to generate a path entangled single photon state. One mode is then amplified using a phase-sensitive amplifier (single-mode squeezer denoted *sqz*) to produce the following state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|1\rangle|\Phi_+\rangle + |0\rangle|\Phi_-\rangle) \quad (1)$$

where $|\Phi_+\rangle = S|0\rangle$ and $|\Phi_-\rangle = S|1\rangle$ are macroscopic

distinct, orthogonal states;

$$|\Phi_+\rangle = \sum_{n=0}^{\infty} \frac{(-\tanh r)^n}{(\cosh r)^{1/2}} \frac{\sqrt{(2n)!}}{2^n n!} |2n\rangle \quad (2)$$

$$|\Phi_-\rangle = \sum_{n=0}^{\infty} \frac{(\tanh r)^n}{(\cosh r)^{3/2}} \frac{\sqrt{(2n+1)!}}{2^n n!} |2n+1\rangle \quad (3)$$

with an average photon number of $\langle \Phi_+ | n | \Phi_+ \rangle = \sinh(r)$ and $\langle \Phi_- | n | \Phi_- \rangle = 2\sinh(r) + \cosh(r)$, respectively. $S = \exp((ra^2 - ra^\dagger)/2)$ is the squeezing operator, r is the squeezing parameter, a is the annihilation operator and $|0\rangle(|1\rangle)$ represent the vacuum (single) photon state. The number of photons of the squeezed states is limited by the degree of squeezing attainable in the optical laboratory. For strongly pumped optical parametric amplifiers, however, this number can be very large as shown in the experiment of Ref. [18]. I also note that the squeezing operation could in principle be replaced by a displacement operation by which the photon numbers can be very large in a practical setting. However, since such an operation solely changes the first moments of the states it does not add extra non-classicality to the state, and cannot be regarded as a cat state.

It is known that the squeezed single photon state and the squeezed vacuum state possess a large fidelity with the odd and even coherent state superpositions states; $|+\rangle = N_+(|\alpha\rangle + |-\alpha\rangle)$ and $|-\rangle = N_-(|\alpha\rangle - |-\alpha\rangle)$ (where $N_{(+,-)}$ are normalization factors), for small values of the coherent state excitations [25]. It means that the proposed state has a great similarity to the following state

$$|\Theta\rangle = \frac{1}{\sqrt{2}}(|1\rangle|+\rangle + |0\rangle|+\rangle) \quad (4)$$

which possess maximal entanglement between a microscopic single photon qubit and a macroscopic coherent state qubit. This state has several applications in quantum information processing [26].

Since the macroscopic states in (3) are orthogonal, the micro-macro state in (1) is maximally entangled. However, for remote preparation of the entangled state, that is, preparing the Fock state components at one site (Alice) and the squeezed states components at another site (Bob), one of the modes must be sent through a lossy channel which will inevitably lead to degradation of the entanglement rendering the state non-maximally entangled. It is however possible to circumvent the propagation losses as the delocalized single photon can be heralded at a distance using the method outlined in ref.[27–29]: The generation of the path entangled photon state can be implemented employing two sources of two-mode squeezed states (one at Alice and one at Bob). One mode from each source combines at a symmetric beam splitter, and the measurement of single photon heralds the desired state. The remotely prepared path entangled single photon state is then subsequently squeezed at one site

(e.g. at Bob) to generate the required state. Using this approach maximally entangled micro-macro states can be generated at a distance independent on the losses between the two sites. I note however that the increase in the state purity is traded for a decrease in the generation rate.

The squeezing operation of a single photon as introduced above is identical to subtracting a single photon from a squeezed vacuum state. This leaves open another way of preparing the entangled state in (1) at a distance. The circuit is illustrated in Fig. 1b. The idea is to jointly subtract a single photon from two locally prepared quantum states; a single photon state at Alice's site and a squeezed vacuum state at Bob's site. The joint subtraction is enabled by a beamsplitter and a single photon counter which is described by the non-unitary operation $\sqrt{T}a_A + \sqrt{R}a_B$ where a_A and a_B are the annihilation operators acting on the modes of Alice and Bob, respectively, and $T(R)$ is the transmission (reflection) coefficient of the beam splitter. The transformation reads

$$(\sqrt{T}a_A + \sqrt{R}a_B)S|0\rangle|1\rangle \rightarrow \sqrt{T}a_AS|0\rangle|1\rangle + \sqrt{R}S|0\rangle|0\rangle \quad (5)$$

which is identical to (1) for $T = R = 1/2$ (up to a bit-flip operation). As the purity of the resulting state is independent on the losses of the joint measurement and the channel, the state can be prepared remotely without degradation. However, as pointed out above, the preparation rate will depend on the losses. The generation strategy has the additional practical advantage of using only off-line non-classical devices which means that there is no need of injecting the non-classical state into a non-linear element. Alternatively, an off-line squeezing operations can be implemented using homodyne based electro-optical feedforward [30, 31].

The state in (4) can be generated by jointly subtracting a single photon from a product state comprising a single photon and a superposition of coherent states (using the scheme in Fig. 1b with $|1\rangle N_+(|\alpha\rangle + |-\alpha\rangle)$ as the input). Such a joint subtraction will produce the state

$$(\sqrt{T}a_A + \sqrt{R}a_B)|1\rangle N_+(|\alpha\rangle + |-\alpha\rangle) \quad (6)$$

$$\rightarrow \sqrt{T}|0\rangle|+\rangle + \sqrt{R}\alpha|1\rangle|-\rangle \quad (7)$$

which is maximally entangled for $\sqrt{T} = \alpha\sqrt{R}$ or $R = 1/(1 + \alpha^2)$. This state can be further transformed into $|1\rangle|\alpha\rangle + |0\rangle|-\alpha\rangle$ if a Hadamard transform is made onto mode B. I note that the coherent state Hadamard transform is in general non-unitary but is approximately unitary for large coherent state excitations [32].

Characterizing the entanglement of the micro-macro state has been debated in the literature. In the experiment in Ref.[18], the entanglement was quantified by using a Stokes parameter measurement to measure the polarization degree of freedom of the single photon and a special filter detector to discriminate the two multi-photon states[21]. Homodyne tomography could not be

used to fully characterize the state due to the a-posteriori type of generation scheme. On the contrary, the state generation schemes suggested in this paper are based on heralding (that is, the states are not produced a posteriori), and thus homodyne detection can be used to fully characterize the state. With two-mode homodyne tomography, the full density matrix can be reconstructed and the entanglement can be evaluated[33].

A microscopic single photon qubit can be mapped onto a macroscopic qumode using the micro-macro entangled states proposed in this paper. The entangled state is used in a teleportation protocol with an arbitrary qubit as the input signal; $c_0|0\rangle + c_1|1\rangle$ where c_0 and c_1 are complex numbers. A Bell measurement that projects onto the four Bell states is jointly performed onto the signal and the entangled state, and the outcome is used to perform a unitary transformation onto the remaining part of the entangled state. A full Bell state measurement is in principle possible[34] but only two projections can be obtained with simple linear optics[35, 36]. With this scheme it is possible to make the following transformation

$$c_0|0\rangle + c_1|1\rangle \rightarrow c_0|\phi_1\rangle + c_1|\phi_2\rangle \quad (8)$$

where $|\phi_1\rangle$ and $|\phi_2\rangle$ are the coherent state superpositions in (4), $|+\rangle$, $|-\rangle$, or the squeezed state in (3), $|\Phi_+\rangle$, $|\Phi_-\rangle$, depending on which entangled state is being used for the teleportation. Such an operation enables one to link a qubit processor to a qumode processor.

In conclusion, I have suggested several optical circuits for generating a heralded version of the optical Schrödinger cat state or the micro-macro state. As opposed to the previous proposals and implementations of a polarization based micro-macro state, the suggested state is entangled in its mode degree of freedom and thus simpler to produce. Moreover the proposed scheme is based on heralding which means that the state can be fully characterized with homodyne tomography. Furthermore, it was shown that the state can be used as the resource in a teleporter to map microscopic qubits onto macroscopic qumodes, possibly at remote locations as the entangled state can be efficiently produced at a distance.

Note: During the preparation of this manuscript I got aware of a similar proposal on preparing a local micro-macro state using a squeezing operation [37]. I also got aware of a proposal on using a displacement operation in replacement of the squeezing operation [38] as briefly mentioned in the present paper after eq. (3).

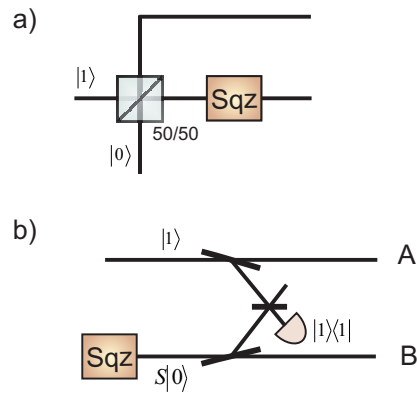


FIG. 1. Schematic setup of the proposed schemes.

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